Bounded Diameter Decompositions in Mathematics and Theoretical Computer Science

Louis Esperet (CNRS, G-SCOP, Grenoble)

20 ans du GDR-I(F)M

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A notion of dimension for metric spaces

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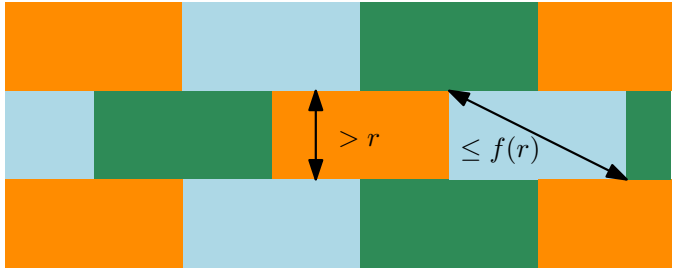

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(source: wikipedia - Hex game)

Asymptotic dimension

Gromov (1993). A metric space X has asymptotic dimension at most d if there is a function f such that for every $r > 0$, X can be covered by clusters of at most $d + 1$ colors, each of diameter at most $f(r)$, and any two sets of the same color are at distance $> r$ apart.

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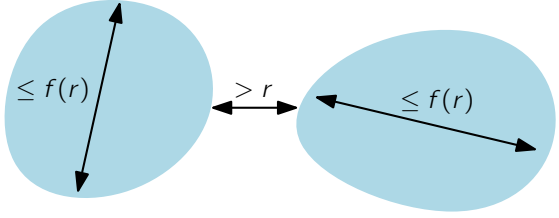

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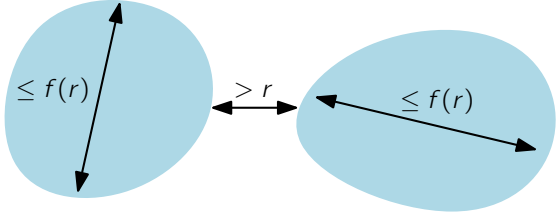

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Quasi-isometry

Two metric spaces (X, d_X) and (Y, d_Y) are quasi-isometric if there is a map $f : X \to Y$ and constants $A_1, A_2, B_1, B_2, C \geq 0$ such that any element of Y is at distance at most C from some element of $f(X)$, and for every $x_1, x_2 \in X$,

$$A_1 \cdot d_X(x_1, x_2) - B_1 \leq d_Y(f(x_1), f(x_2)) \leq A_2 \cdot d_X(x_1, x_2) + B_2.$$

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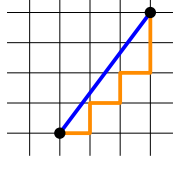
Example. $X =$ the 2-dimensional grid, $Y = \mathbb{R}^2$, f : the grid $\to \mathbb{Z}^2$.

$$\frac{1}{\sqrt{2}} d_X(x_1, x_2) \leq d_Y(f(x_1), f(x_2)) \leq d_X(x_1, x_2).$$



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Observation

Quasi-isometric spaces have the same asymptotic dimension.

Geometric group theory

Given a finitely generated group G and a finite symmetric set of generators S, the Cayley graph of (G, S) has vertex set x and an edge between any element $x \in G$ and any element $xs \in G$ $(s \in S)$.

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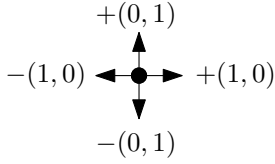

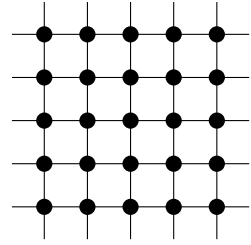

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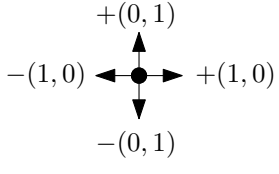

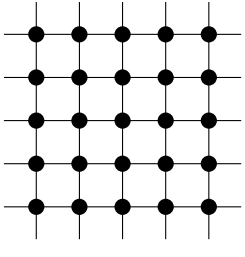

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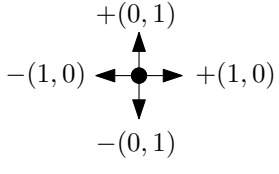

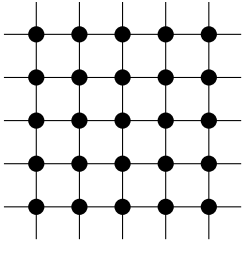

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Asymptotic dimension is a group invariant !

A convenient rephrasing

A graph G has asymptotic dimension at most d if there is a function f such that for every $r > 0$, G can be covered by clusters of at most $d + 1$ colors, each of diameter at most $f(r)$, and any two sets of the same color are at distance $> r$ apart.

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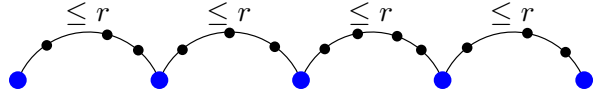

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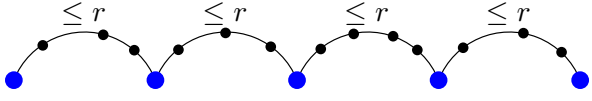

Equivalent definition

A graph G has asymptotic dimension at most d if there is a function f such that for every $r > 0$, G has a $(d + 1)$-coloring, in which each monochromatic sequence as above has diameter at most $f(r)$ in G.

Some results in graph theory

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Theorem (Bonamy, Bousquet, Esperet, Groenland, Liu, Pirot, and Scott 2020)

For every t, K_t-minor-free graphs have asymptotic dimension at most 2.

Sparse partitions

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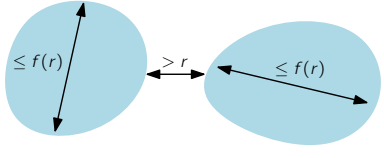

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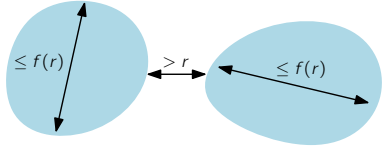

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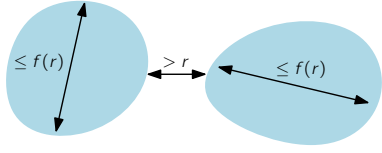

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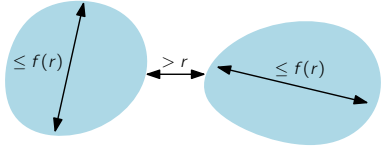

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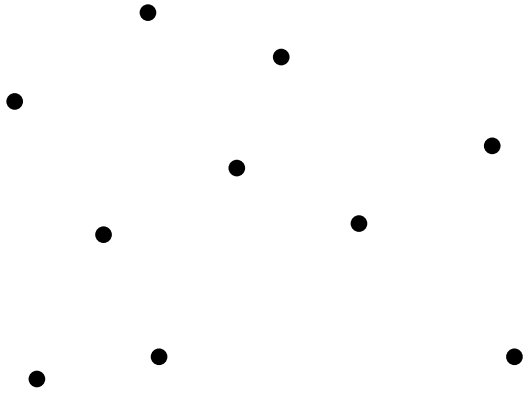
When c and d are small, several combinatorial optimisation problems can be approximated efficiently.

Universal Traveling Salesman Problem



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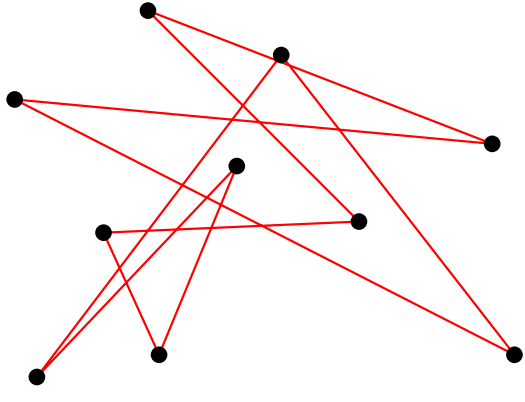

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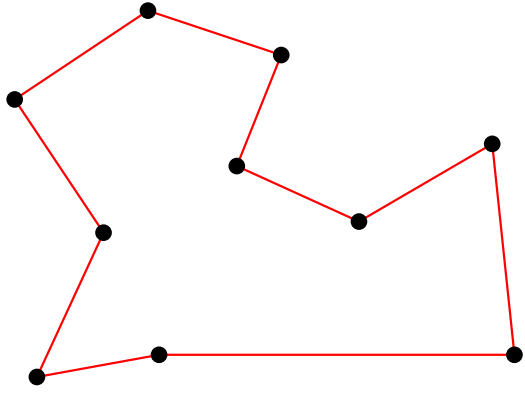

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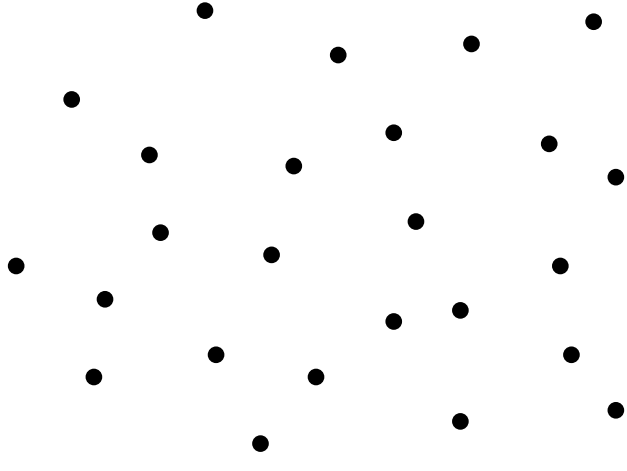

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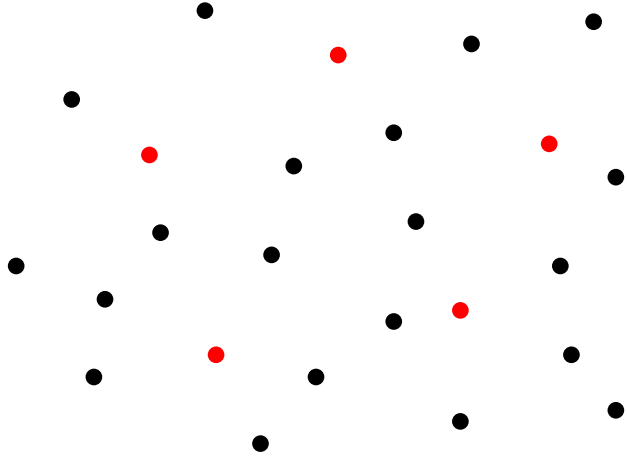

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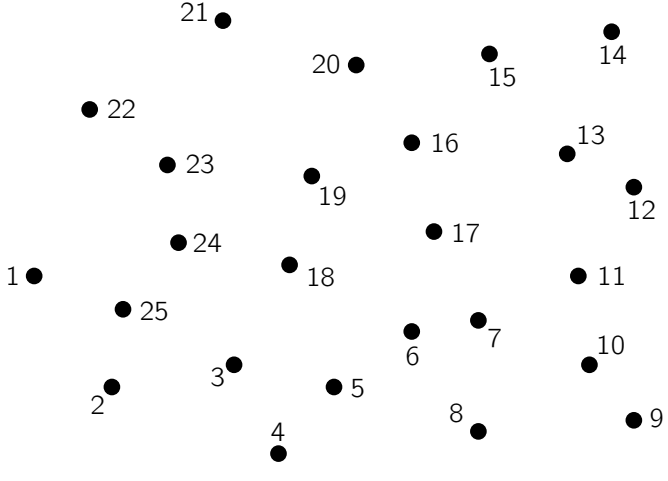

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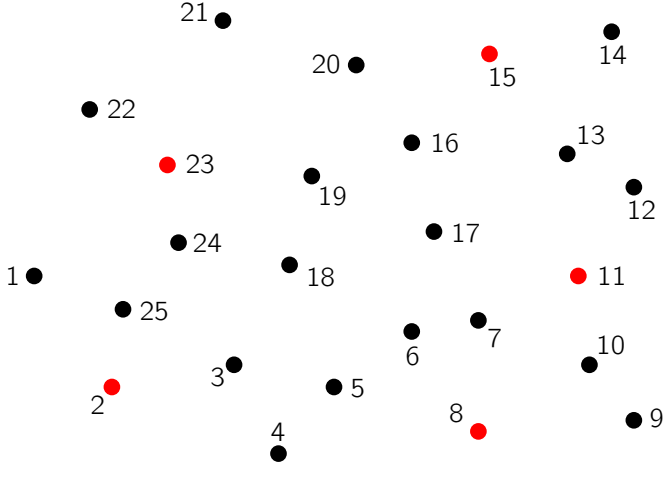

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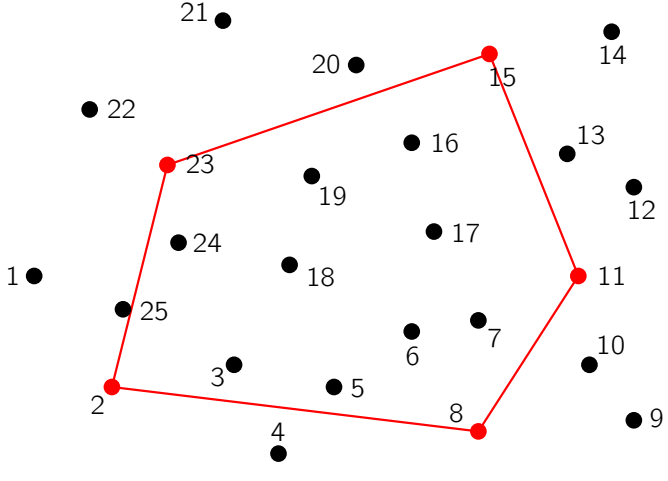

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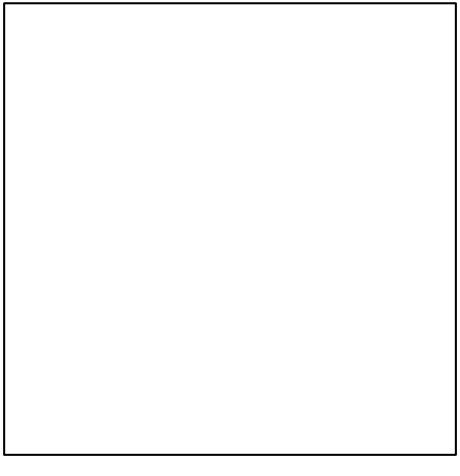

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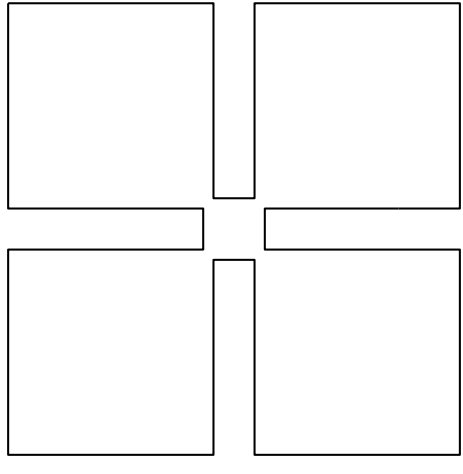

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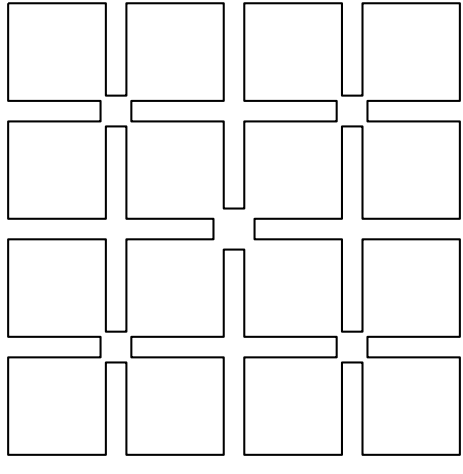

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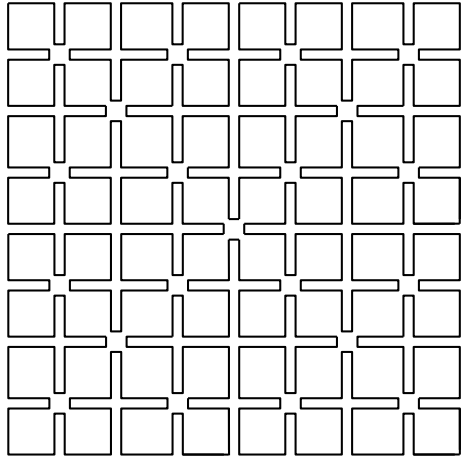

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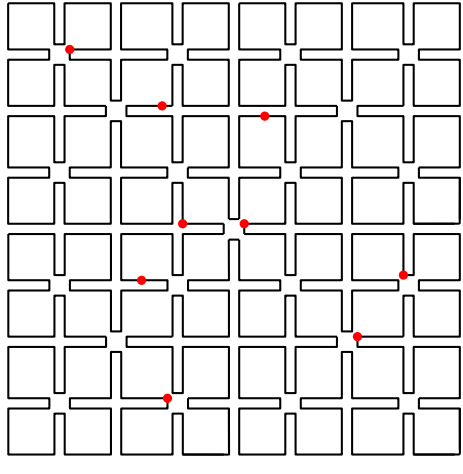

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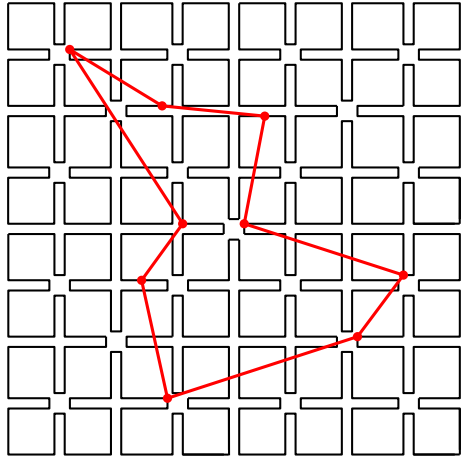

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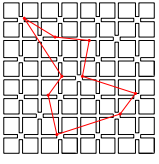

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Ordering n points in the plane following the space-filling curve gives a $O(\log n)$-approximation for the TSP for any subset of the points.

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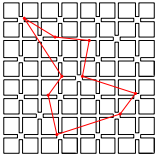

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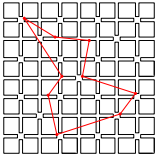

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The same holds for several other universal variants of classical combinatorial optimization problems (Universal Steiner Tree, ...)

Network decompositions

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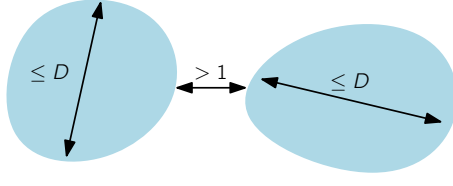
Taking $r = 1$, for any class of graphs of bounded asymptotic dimension, there exist C and D such that every graph in the class has a partition into clusters of C colors, such that neighboring clusters have different colors and every cluster has diameter at most D.



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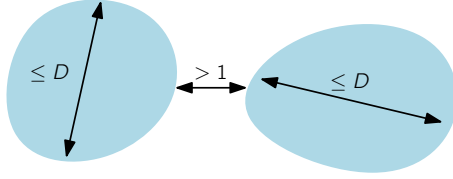

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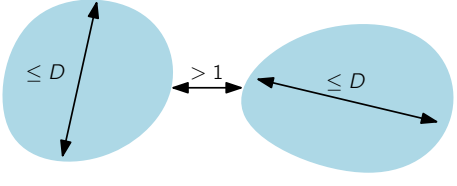

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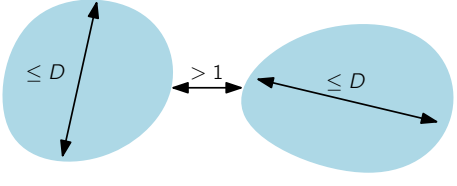

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This has proved to be a central tool in distributed computing.

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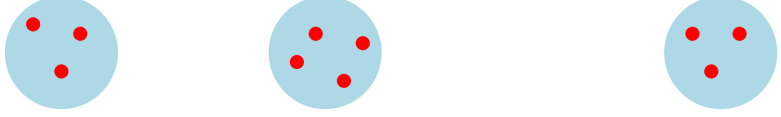

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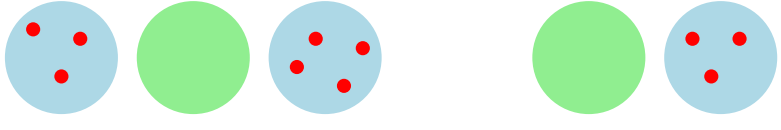

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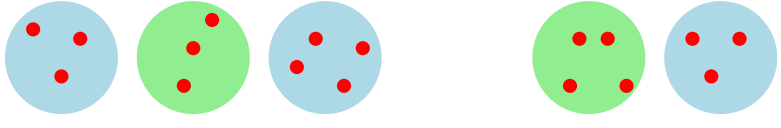

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Theorem (Ghaffari and Grunau 2024)

A network decomposition with $O(\log n)$ colors and diameter $O(\log n)$ can be computed deterministically in $O(\log^2 n)$ rounds.

Generating random spanning trees

Theorem (Broder 1989 and Aldous 1990)

Perform a random walk W in G, starting at some vertex s, and stop when all the vertices of G have been visited. For each $v \neq s$, consider the first edge incident to v encountered by W. The union of these edges is a uniform random spanning tree of G.

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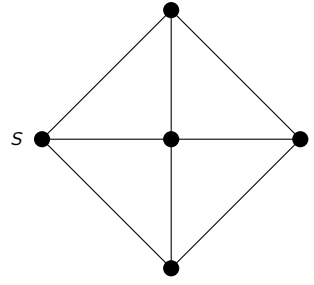

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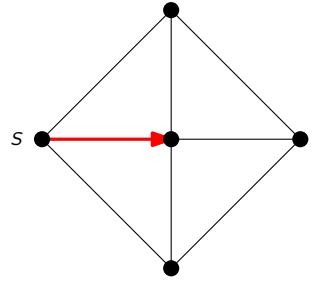

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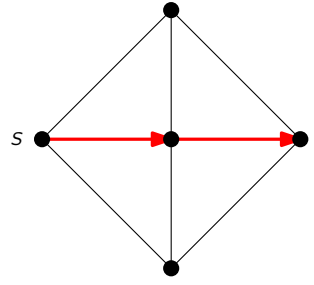

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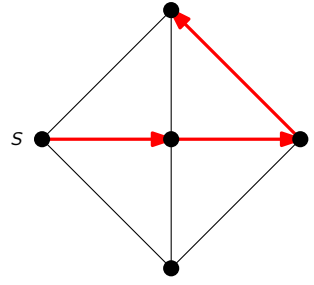

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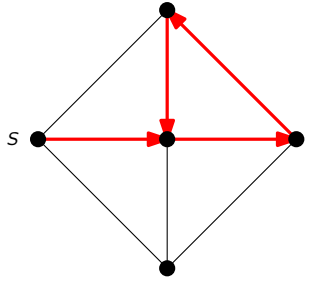

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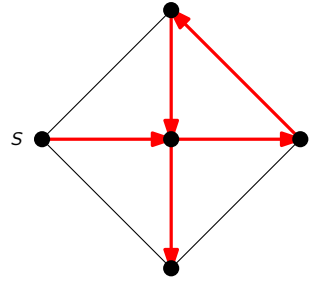

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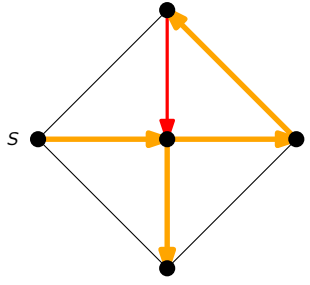

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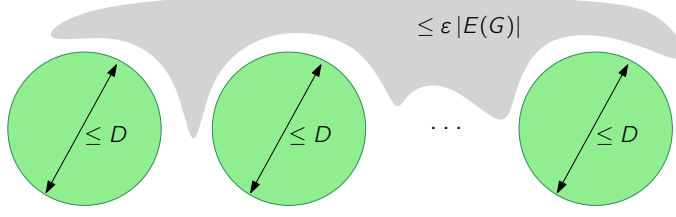

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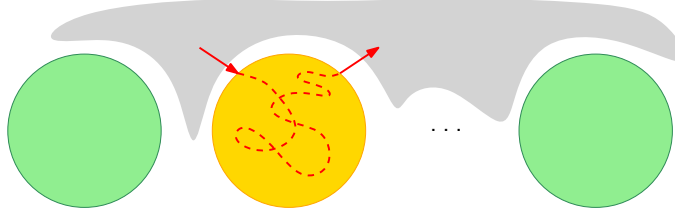

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Schild (2018), and Anari, Liu, Oveis Gharan, Vinzant, and Vuong (2021) obtained new algorithms running in nearly linear time.

Conclusion

We have seen that low diameter decompositions had been studied and used in several areas, mostly independently between pure math and TCS:

- Geometric group theory
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- Distributed computing
- Approximation algorithms / Combinatorial optimization
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In my opinion this illustrates the importance of having organizations & meetings bringing together researchers from very diverse backgrounds in the math-TCS spectrum, and facilitating the flow of ideas between the fields.